Mark Scheme (Results)

Summer 2019

Pearson Edexcel GCE
In Mathematics (9MA0_32) Mechanics

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk. Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus.

Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

Summer 2019
Publications Code 9MA0_03_1906_MS

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS
General Instructions for Marking

1. The total number of marks for the paper is 50.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\surd$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $*$ The answer is printed on the paper
- The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

General Principles for Mechanics Marking

(But note that specific mark schemes may sometimes override these general principles)

- Rules for M marks: correct no. of terms; dimensionally correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.

- Omission or extra g in a resolution is an accuracy error not method error.

- Omission of mass from a resolution is a method error.

- Omission of a length from a moments equation is a method error.

- Omission of units or incorrect units is not (usually) counted as an accuracy error.

- DM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.

- Any numerical answer which comes from use of g = 9.8 should be given to 2 or 3 SF.

- Use of g = 9.81 should be penalised once per (complete) question.

  N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *once* per complete question. However, premature approximation should be penalised every time it occurs.

- Marks must be entered in the same order as they appear on the mark scheme.

- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),……then that working can only score marks for that part of the question.

- Accept column vectors in all cases.

- Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft

- Mechanics Abbreviations

  M(A) Taking moments about A.

  N2L Newton's Second Law (Equation of Motion)

  NEL Newton's Experimental Law (Newton's Law of Impact)

  HL Hooke's Law

  SHM Simple harmonic motion

  PCLM Principle of conservation of linear momentum

  RHS, LHS Right hand side, left hand side.

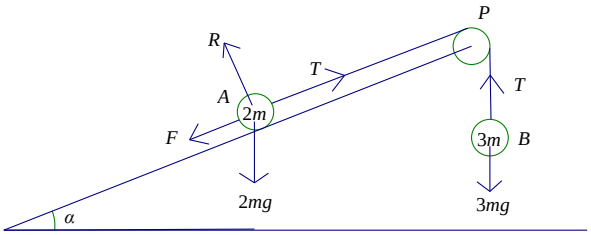
Question	Scheme	Marks	AO
1(a)	Differentiate **v**	M1	1.1a
	$(\mathbf{a}=)6\mathbf{i}-\frac{15}{2}t^{\frac{1}{2}}\mathbf{j}$	A1	1.1b
	$=6\mathbf{i}-15\mathbf{j}$ (m s^{-2})	A1	1.1b
		(3)	
1(b)	Integrate **v**	M1	1.1a
	$(\mathbf{r}=)(\mathbf{r}_0)+3t^2\mathbf{i}-2t^{\frac{5}{2}}\mathbf{j}$	A1	1.1b
	$=(-20\mathbf{i}+20\mathbf{j})+(48\mathbf{i}-64\mathbf{j})=28\mathbf{i}-44\mathbf{j}$ (m)	A1	2.2a
		(3)	
		(6)	

Marks		Notes
		N.B. Accept column vectors throughout and condone missing brackets in working but they must be there in final answers
1a	M1	Use of $\mathbf{a}=\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t}$ with attempt to differentiate (both powers decreasing by 1) M0 if **i**'s and **j**'s omitted and they don't recover
	A1	Correct differentiation in any form
	A1	Correct and simplified. Ignore subsequent working (ISW) if they go on and find the magnitude.
1b	M1	Use of $\mathbf{r}=\int\mathbf{v}\mathrm{d}t$ with attempt to integrate (both powers increasing by 1) M0 if **i**'s and **j**'s omitted and they don't recover
	A1	Correct integration in any form. Condone $\mathbf{r}_0$ not present
	A1	Correct and simplified.

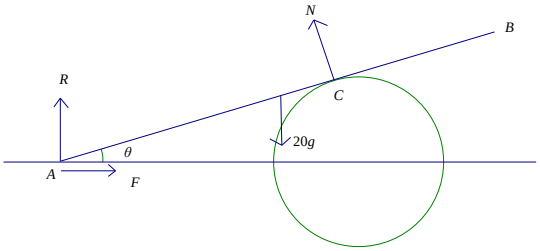
Question	Scheme	Marks	AO
2(a)	$(\mathbf{v}=)\mathbf{C}+(2\mathbf{i}-3\mathbf{j})t$	M1	3.1a
	$(\mathbf{v}=)(-\mathbf{i}+4\mathbf{j})+(2\mathbf{i}-3\mathbf{j})t$	A1	1.1b
	$\frac{4-3T}{-1+2T}=\frac{-4}{3}$ oe	M1	3.1a
	$T=8$	A1	1.1b
		(4)	
(b)	$(\mathbf{s}=)\mathbf{C}t+(2\mathbf{i}-3\mathbf{j})\frac{1}{2}t^2$ (+ **D**)	M1	3.1a
	$(\mathbf{s}=)(-\mathbf{i}+4\mathbf{j})t+\frac{1}{2}(2\mathbf{i}-3\mathbf{j})t^2$ (+ **D**)	A1	1.1b
	$AB=\sqrt{12^2+8^2}$ **N.B. Beware you may see 4(2i – 3j) which leads to** $\sqrt{(8^2+12^2)}$ this is M0A0M0A0.	M1	3.1a
	$=4\sqrt{13}(=14.422051....)$ (m)	A1**cso**	1.1b
		(4)	
		(8)	

Marks		Notes
2a	M1	Use of $\mathbf{v}=\mathbf{u}+\mathbf{a}t$ **OR** integration to give an expression of the form $\mathbf{C}+(2\mathbf{i}-3\mathbf{j})t$ **, where C is a non-zero constant <u>vector</u>** M0 if **u** and **a** are reversed Condone use of **a** = (2**i** + 3**j)** for this M mark
	A1	Any correct unsimplified expression seen or implied
	M1	Correct use of ratios, <u>using a velocity vector</u> (must be using $\frac{-4}{3}$) to give equation <u>in *T* only</u> M0 if they equate $4-3T=-4$ and/or $-1+2T=3$ and therefore M0 if they then divide to produce their equation
	A1	Correct only
		N.B. (i) Can score the second M1A1 if they get $T = 8$, using a calculator to solve two simultaneous equations, but if answer is wrong, and no equation in *T* only, second M0 (ii) Can score M1A1 M1A1 if they get $T = 8$, using trial and error, but if they don't get $T = 8$, can only score max M1A1M0A0

2b	M1	Use of $\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ with **a = (2i – 3j)** **OR** integration to give an expression of the form $\mathbf{C}t + (2\mathbf{i} - 3\mathbf{j})\frac{1}{2}t^2$ **, where C is their non-zero constant <u>vector</u> from (a)** Condone use of **a** = (2**i +** 3**j)** for this M mark **OR** any other complete method using vector **suva***t* equations
	A1	Correct unsimplified expression seen or implied
	M1	Use of $t = 4$ in their **s** (which must be a **displacement vector**) and then Pythagoras with the root sign **N.B.** This M mark can be implied by a correct answer, otherwise we need to see Pythagoras used, with the root sign, for the M mark.
	A1**cso**	Any surd form or 14 or better

Question	Scheme	Marks	AO
3(a)			
	$R = 2mg\cos\alpha$	B1	3.4
	$F = \frac{2}{3}R$	B1	1.2
	Equation of motion for A:	M1	3.3
	$T - F - 2mg\sin\alpha = 2ma$	A1	1.1b
	Equation of motion for B:	M1	3.3
	$3mg - T = 3ma$	A1	1.1b
	Complete strategy to find an equation in T, m and g only.	M1	3.1b
	$T = \frac{12mg}{5}$ *	A1*	2.2a
		(8)	
(b)	$(F_{\max} =)\frac{16mg}{13} > \frac{10mg}{13}$	M1	2.1
	…… so A will not move.	A1	2.2a
		(2)	
(c)	• Extensible string • Weight of string • Friction at pulley e.g. rough pulley • Allow for the dimensions of the blocks e.g. "Do not model blocks as particles"; "(include) air resistance";"include rotational effects of forces on blocks i.e. spin"	B1 B1	3.5c 3.5c
		(2)	
		(12)	

Marks		Notes
3a	B1	Normal reaction between *A* and the plane seen or implied, $\cos\alpha$ does not need to be substituted.
	B1	$F = \frac{2}{3}R$ seen or implied anywhere, including part (b)
	M1	Form an equation of motion for *A*. Must include all relevant terms. Must be the correct mass but condone consistent missing *m*'s. Condone sign errors and sin/cos confusion
	A1	Correct unsimplified equation (*F* does not need to be substituted). Allow consistent use of $(-a)$ **N.B**. If $T - 2mg = 2ma$ is seen with no working, M0A0 unless both B1 marks have been scored.
	M1	Form an equation of motion for *B*. Must be the correct mass on RHS but condone consistent missing *m*'s. Condone sign errors and sin/cos confusion.
	A1	Correct unsimplified equation (*F* does not need to be substituted). Allow consistent use of $(-a)$
		N.B. Allow the 'whole system' equation to replace the equation for *A* or *B*. $3mg - F - 2mg\sin\alpha = 5ma$ Must be the correct mass on RHS but condone consistent missing *m*'s. Condone sign errors and sin/cos confusion.
	M1	Complete method to give an equation in *T*, *m* and *g* only. **N.B.** Allow θ in the equation if they have defined what θ is: e.g. $\theta = \tan^{-1}(\frac{5}{12})$ This is an <u>independent</u> mark but they must have two simultaneous equations in *T* and *a* unless one of the equations is the whole system equation in which case one equation will be in *T* and *a* and the other equation will be in *a* only.
	A1*	Obtain the **given answer** from correct working using EXACT trig ratios. (not available if using a decimal angle)
3b	M1	Comparison of their $F_{\max}$ $(\frac{2}{3}R)$ and their component of weight down the slope, must be comparing numerical values. oe e.g. if they consider the difference **N.B.** Allow comparison of μ and $\tan\alpha$ with numerical values
	A1	Correctly justified conclusion and no errors seen **N.B.** If they equate their difference to an '*ma*' term then A0
3c	B1 B1	Deduct 1 mark for each extra (more than 2) incorrect answer up to a maximum of 2 incorrect answers. Ignore extra correct answers. e.g. two correct, one incorrect B1 B0 one correct, one incorrect B1 B0 one correct, two incorrect B0 B0 Ignore incorrect reasons or consequences. Ignore any mention of wind or a general reference to friction.

Question	Scheme	Marks	AO
4(a)	Drum **smooth**, or no friction, (therefore reaction is perpendicular to the ramp)	B1	2.4
		(1)	
(b)	**N.B.** In (b), for a moments equation, if there is an extra $\sin\theta$ or $\cos\theta$ on a length, give M0 for the equation e.g. M(A): $20g\times4\cos\theta=5N\sin\theta$ would be given M0A0		
			
	Possible equns $(\nearrow): F\cos\theta+R\sin\theta=20g\sin\theta$ $(\nwarrow): N+R\cos\theta=20g\cos\theta+F\sin\theta$ $(\uparrow) R+N\cos\theta=20g$ $(\rightarrow): F=N\sin\theta$ M(A): $20g\times4\cos\theta=5N$ M(B): $3N+R\times8\cos\theta=F\times8\sin\theta+20g\times4\cos\theta$ M(C): $R\times5\cos\theta=F\times5\sin\theta+20g\times\cos\theta$ M(G): $R\times4\cos\theta=F\times4\sin\theta+N$	M1 A1 M1 A1 M1 A1	3.3 1.1b 3.4 1.1b 3.4 1.1b
	(The values of the 3 unknowns are: N = 150.528; F = 42.14784; R = 51.49312)		
	Alternative 1: using cpts along ramp (X) and perp to ramp(Y) Possible equations: $(\nearrow): X=20g\sin\theta$ $(\nwarrow): Y+N=20g\cos\theta$ $(\uparrow): X\sin\theta+Y\cos\theta+N\cos\theta=20g$ $(\rightarrow): X\cos\theta=Y\sin\theta+N\sin\theta$ M(A): $20g\times4\cos\theta=5N$ M(B): $20g\times4\cos\theta=8Y+3N$ M(C): $20g\times\cos\theta=5Y$ M(G): $4Y=N\times1$	M1 A1 M1 A1 M1 A1	3.3 1.1b 3.4 1.1b 3.4 1.1b
	(The values of the 3 unknowns are: N = 150.528; X = 54.88; Y = 37.632)		

	Alternative 2: using horizontal cpt (*H*) and cpt perp to ramp (*S*)		
	$(\nearrow): H\cos\theta = 20g\sin\theta$	M1	3.3
	$(\nwarrow): S + N = H\sin\theta + 20g\cos\theta$	A1	1.1b
	$(\uparrow): S\cos\theta + N\cos\theta = 20g$	M1	3.4
	$(\rightarrow): H = S\sin\theta + N\sin\theta$	A1	1.1b
	$\text{M}(A): 20g \times 4\cos\theta = 5N$	M1	3.4
	$\text{M}(B): 20g \times 4\cos\theta + H \times 8\sin\theta = 8S + 3N$	A1	1.1b
	$\text{M}(C): 20g \times \cos\theta + H \times 5\sin\theta = 5S$		
	$\text{M}(G): 4S = N \times 1 + H \times 4\sin\theta$		
	(The values of the 3 unknowns are: N = 150.528; H = 57.1666…; S = 53.638666…)		
	Solve their 3 equations for F and R **OR** X and Y **OR** H and S	M1	1.1b
	$\lvert\text{Force}\rvert = \sqrt{R^2 + F^2}$ Main scheme **OR** $= \sqrt{X^2 + Y^2}$ Alternative 1 **OR** $= \sqrt{(H^2 + S^2 - 2HS\cos(90^o - \theta)}$ Alternative 2	M1	3.1b
	Magnitude = 67 or 66.5 (N)	A1	2.2a
		(9)	
(c)	Magnitude of the normal reaction (at *C*) will **decrease**.	B1	3.5a
		(1)	
		(11)	

Marks		Notes
4a	B1	Ignore any extra incorrect comments.
		Generally 3 independent equations required so **at least one moments equation.**: M1A1M1A1M1A1. More than 3 equations, give marks for the best 3. For each: M1 All terms required. Must be dimensionally correct so if a length is missing from a moments equation it's M0 Condone sin/cos confusion. A1 For a correct equation (trig ratios do not need to be substituted and allow e.g. cos(24/25) if they recover Enter marks on ePEN in order in which equations appear. **N.B.** If reaction at *C* is not perpendicular to the ramp, can only score marks for M(*C*) Allow use of (μR) for *F*
4b	M1	All terms required. Must be dimensionally correct. Condone sin/cos confusion.
	A1	Correct unsimplified equation
	M1	All terms required. Must be dimensionally correct. Condone sin/cos confusion.
	A1	Correct unsimplified equation
	M1	All terms required, dim correct, condone sin/cos confusion
	A1	Correct unsimplified equation
		N.B. They can find *F* and *R* using only TWO equations, the 1st and 7th in the list. Mark the better equation as M2A2 (-1 each error). Mark the second equation as M1A1
Alt 1	M1	All terms required. Must be dimensionally correct. Condone sin/cos confusion.
	A1	Correct unsimplified equation
	M1	All terms required. Must be dimensionally correct. Condone sin/cos confusion.
	A1	Correct unsimplified equation
	M1	All terms required. Must be dimensionally correct. Condone sin/cos confusion.
	A1	Correct unsimplified equation
		N.B. They can find *X* and *Y* using only TWO equations, the 1st and 7th in the list. Mark the better equation as M2A2 (-1 each error). Mark the second equation as M1A1
Alt 2	M1	All terms required. Must be dimensionally correct. Condone sin/cos confusion.
	A1	Correct unsimplified equation
	M1	All terms required. Must be dimensionally correct. Condone sin/cos confusion.

	A1	Correct unsimplified equation
	M1	All terms required. Must be dimensionally correct.
	A1	Correct unsimplified equation
		N.B. They can find *H* and *S* using only TWO equations, the 1st and 7th in the list. Mark the better equation as M2A2 (-1 each error). Mark the second equation as M1A1
	M1	Substitute for trig and solve for their two cpts. This is an independent mark but must use 3 equations (unless it's the special case when 2 is sufficient)
	M1	Use Pythagoras to find magnitude (this is an independent M mark but must have found a value for *F* (or *X*) and a value for *R* (or *Y*)) **OR** a complete method to find magnitude e.g. cosine rule but must have found a value for *H* and a value for *S*
	A1	Correct answer only
	B1	Ignore reasons

Question	Scheme	Marks	AO
	In this question mark parts (a) and (b) together.		
5(a)	Horizontal speed $= 20\cos 30°$	B1	3.4
	Vertical velocity <u>at $t = 2$</u>	M1	3.4
	$= 20\sin 30° - 2g$	A1	1.1b
	$\theta = \tan^{-1}\left(\pm\dfrac{9.6}{10\sqrt{3}}\right)$	M1	1.1b
	Speed $= \sqrt{100\times 3 + 9.6^2}$ or e.g. speed $= \dfrac{9.6}{\sin\theta}$	M1	1.1b
	19.8 or 20 $\left(\text{m s}^{-1}\right)$ at 29.0° or 29° to the horizontal oe	A1	2.2a
		(6)	
(b)	Using sum of horizontal distances $= 50$ at $t = 2$	M1	3.3
	$(u\cos\theta)\times 2 + (20\cos 30°)\times 2 = 50$ $(u\cos\theta = 25 - 20\cos 30°)$	A1	1.1b
	Vertical distances equal	M1	3.4
	$\Rightarrow (20\sin 30°)\times 2 - \dfrac{g}{2}\times 4 = (u\sin\theta)\times 2 - \dfrac{g}{2}\times 4$ $(20\sin 30° = u\sin\theta)$	A1	1.1b
	Solving for both θ and u	M1	3.1b
	$\theta = 52°$ or better (52.47756849….°) $u = 13$ or better (12.6085128…)	A1	2.2a
		(6)	
(c)	It does not take account of the fact that they are not particles (moving freely under gravity) It does not take account of the size(s) of the balls It does not take account of the spin of the balls It does not take account of the wind g is not exactly 9.8 m s^{-2} **N.B.** If they refer to the mass or weight of the balls give B0	B1	3.5b
		(1)	
		(13)	

Marks		**Notes**		
5a	B1	Seen or implied, possibly on a diagram		
	M1	Use of $v = u + at$ or any other complete method <u>using $t = 2$</u> Condone sign errors and sin/cos confusion.		
	A1	Correct unsimplified equation in v or v^2		
	M1	Correct use of trig to find a relevant angle for the direction. Must have found a horizontal and a vertical velocity component		
	M1	Use Pythagoras or trig to find the magnitude Must have found a horizontal and a vertical velocity component		
	A1	Or equivalent. Need magnitude **and** direction stated or implied in a diagram. (0.506 or 0.51 rads)		
5b	M1	First equation,in terms of u and θ (could be implied by subsequent working), using the horizontal motion with $t = 2$ used Condone sign errors and sin/cos confusion		
	A1	Correct unsimplified equation – any equivalent form		
	M1	Second equation, in terms of u and θ (could be implied by subsequent working), using the vertical motion – equating distances or just vertical components of velocities. Condone sign errors and sin/cos confusion		
	A1	Correct unsimplified equation – any equivalent form		
	M1	Complete strategy: all necessary equations formed and solve for u and θ **N.B.** This is an independent method mark but can only be earned if 50 m has been used in their solution.		
	A1	Both values correct. (Here we accept 2SF or better, since the g's cancel) Allow radians for θ: 0.92 or better (0.915906..) rads.		
5c	B1	Any factor **related to the model** as stated in the question. Penalise incorrect extras but ignore consequences e.g. '*AB* (or the ground) is not horizontal' should be penalised or 'they do not move in a vertical plane' should be penalised		

Pearson Education Limited. Registered company number 872828
with its registered office at 80 Strand, London, WC2R 0RL, United Kingdom